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$H\infty$ observer-based sliding mode control for uncertain discrete-time singularly perturbed systems

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Abstract

This paper studies the sliding mode control (SMC) problem of discrete-time singularly perturbed systems (DTSPSs) with external perturbations and nonlinear inputs. First, to estimate the state of the original system, a novel state observer is constructed. Under this condition, the observer error system is ensured to be input-to-state stable (ISS) and satisfies the given $H\infty$ performance index. Then, an appropriate sliding mode control law is proposed to satisfy the reachability condition. On this basis, the ISS criterion for the sliding mode dynamics (SMDs) is given, and the controller gain matrix is also obtained. In addition, a feasible algorithm is given to compute the upper bound for the small parameter. Finally, two numerical examples are given to demonstrate the effectiveness of the proposed method.

Keywords: $H\infty$ observer-based control; Discrete-time singularly perturbed systems; Sliding mode control; Linear matrix inequality

1 Introduction

Singularly perturbed systems (SPSs) are an important research topic in the field of automatic control and applied mechanics, physics, astronomy and other domains [1–5]. However, in practical engineering problems, uncertainty and external disturbances frequently induce instability and are usually considered as the main factors for system performance degradation [6, 7]. So, the discussion on the topic is meaningful.

As is well known, the control of the SPSs has received considerable attention in the past decades [8–14], while the SMC is one of most effective robust control policies for many kinds of systems, which gives the SMC great research value in the field of industry, mechanical engineering, and aerospace industry [15–19]. Yang and Che in [20] constructed a stable sliding surface based on the reachability condition and derived a criterion by linear matrix inequality (LMI) technology and ε bound estimation method. The criterion guaranteed that the closed-loop system is internally exponentially stable. Reference [21] investigated the slow and fast subsystems using the sliding mode control theory. Then, a sufficient condition was provided such that the full-order closed-loop SPS is asymptotically stable. Recently, the relevant results have been extended to stochastic control [22, 23].

In [22], the finite-time bounded problem for stochastic nonlinear singular systems is addressed, in which an appropriate sliding mode function is constructed such that the system exhibits the finite-time bounded stability with the prescribed $H\infty$ performance.

The discrete-time control has been intensified in recent years due to the control implemented by using computers [24, 25]. The authors of [25] studied simplex SMC for a linear DTSPS by modified simplex method, in which a sliding mode control law is constructed to guarantee reachability. In addition, the upper bound of the parameter is obtained by generalized Nyquist plot. Nevertheless, the study of SMC for DTSPSs is far from complete and more work needs to be done.

On the other hand, it is worth noting that the trajectories of system state are not along stable sliding surface strictly, but move up and down the stable sliding surface and tend to equilibrium point. This phenomenon is an inherent disadvantage for discrete-time systems and is influenced by external factors. The approach of observer is adopted to weaken up the chattering phenomenon. The main features of this method are to reduce the external disturbances and to make up for uncertainty. So, designing an appropriate state observer is a key. Many efforts have been made to solve this problem. See [26–32]. References [26–28] mentioned the observer-based SMC for SPSs. The observer-based SMC for uncertain systems is involved in [29, 30]. Among them, a way for designing state observer and sliding mode control law is provided in [28], and the criteria for stability of observer error system and closed-loop system were presented based on LMI. However, few existing results focus on how to employ observer to investigate the SMC problem for uncertain DTSPSs with input nonlinearity. So, further study needs to be carried out for the problem.

In view of the works above, this paper addresses the $H\infty$ sliding mode control for uncertain DTSPSs with external disturbances and input nonlinearity. First, to estimate the state of the original system, a novel state observer is constructed. Under this condition, with the help of Lyapunov method, the observer error system is ensured to be input-to-state stable (ISS) and satisfies the given $H\infty$ performance index. Then, an appropriate sliding mode control law is proposed to satisfy the reachability condition. On this basis, the ISS criterion for the sliding mode dynamics (SMDs) is given and the controller gain matrix is also obtained. Compared to the previous works, the advantages can be summarized as follows: (1) To overcome the difficulty caused by the small parameter in deriving the sliding mode dynamics, a novel sliding mode surface with the SMC gain matrix is constructed to ensure the desired property of the sliding mode dynamics; (2) The method presented in this paper is more convenient in the application, despite the fact that the system considered here is more complicated. The reason is that the proposed sufficient condition does not depend on the equality constraint or Ricatti equations, which reduces the computation cost imposed by the iterative algorithm; (3) A workable way for addressing the upper bound is provided, which can be converted into an optimal problem. It is noticed that the prescribed upper bound is required in many references when solving, which brings great difficulties to the practical operation. The reason is that it is difficult to choose an appropriate upper bound to guarantee the solvability. However, this case has been avoided in this paper.

2 Problem formulation

Consider the uncertain DTSPSs with external disturbance and input nonlinearity

$$x(k+1) = (A_\varepsilon + \Delta A)x(k) + B_{u\varepsilon}\varphi(u) + D_{w\varepsilon}w(k), \tag{1}$$

$$y(k) = Cx(k), \tag{2}$$

where $x(k) = (x_1^T(k), x_2^T(k))^T$ is the system state vector; $\varphi(u)$ is the nonlinear input; $w(k) \in \mathbb{R}^p$ is an external disturbance input vector; $y(k) \in \mathbb{R}^m$ is the system output vector; $\varepsilon > 0$ is a singular perturbation parameter; ΔA is a uncertain matrix with appropriate dimension; A_ε, $B_{u\varepsilon}$, $D_{w\varepsilon}$, C are known and satisfy the following definition:

$$A_\varepsilon = E_0 + E_\varepsilon A, \qquad B_{u\varepsilon} = E_\varepsilon B_u, \qquad B_{w\varepsilon} = E_\varepsilon B_w,$$

where A, B_u, C, D_w, E_ε and E_0 are matrices with appropriate dimensions

$$E_0 = \begin{pmatrix} I & O \\ O & O \end{pmatrix}, \qquad E_\varepsilon = \begin{pmatrix} \varepsilon I & O \\ O & I \end{pmatrix}, \qquad A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$
$$B_u = \begin{pmatrix} B_{u1} \\ B_{u2} \end{pmatrix}, \qquad D_w = \begin{pmatrix} D_{w1} \\ D_{w2} \end{pmatrix}, \qquad C = \begin{pmatrix} C_1 & C_2 \end{pmatrix}.$$

Assumption 2.1 Matrix B_u is of full column remark.

Assumption 2.2 The uncertain matrix ΔA is assumed to satisfy the following admissible form:

$$\Delta A = MF(k)N,$$

where M, N are real constant matrices, $M = (M_1^T\ M_2^T)^T$, $N = (N_1\ N_2)$, the time-varying matrix function $F(k)$ satisfy

$$F^T(k)F(k) \le I. \tag{3}$$

Assumption 2.3 Nonlinear input function $\varphi(u)$ satisfies the following property:

$$u^T(k)\big(\alpha u(k) - \varphi(u)\big) \le 0, \quad u(k) \in \mathbb{R}^q,$$

where α is considered as a nonzero positive constant, and $\varphi(0) = 0$.

The $H\infty$ performance index is chosen as

$$J(k) = \sum_{k=0}^{\infty} z^T(k)z(k) - \gamma^2 w^T(k)w(k). \tag{4}$$

Some basic lemmas are given before proceeding.

Lemma 2.1 ([33]) *System $x(k+1) = f(x(k), u(k))$ is ISS, if there exist a class $\mathcal{K}_\infty$ function α_1, α_2 and a class $\mathcal{K}$ function ρ, and continuous positive definite functions $W(x)$ in $\mathbb{R}^n$ satisfy the following inequalities*:

$$\alpha_1(\|x\|) \le V(x) \le \alpha_2(\|x\|),$$
$$V(x(k+1)) - V(x(k)) \le -W(x(k)), \quad \|x\| \ge \rho(\|u\|),$$

where $V : \mathbb{R}^n \to \mathbb{R}$ is continuous differentiable.

Lemma 2.2 ([17]) *Let Σ_1 and Σ_2 be constant matrices with appropriate dimensions. Then for any $F(k)$ satisfying* (5) *and a scalar $\delta > 0$, the following inequality holds*

$$\Sigma_1 F(k)\Sigma_2 + (\Sigma_1 F(k)\Sigma_2)^T \le \delta\Sigma_1\Sigma_1^T + \delta^{-1}\Sigma_2^T\Sigma_2.$$

Lemma 2.3 ([10]) *Let Π_1, Π_2, and Π_3 be symmetric matrices with appropriate dimensions, η_0 be nonnegative scalar, then $\Pi_1 + \eta\Pi_2 + \eta^2\Pi_3 \ge 0$, $\forall \eta \in (0, \eta_0]$, if the following conditions hold*:

(1) $\Pi_1 \ge 0$;

(2) $\Pi_1 \ge 0, \quad \Pi_1 + \eta_0\Pi_2 \ge 0$;

(3) $\Pi_1 \ge 0, \quad \Pi_1 + \eta_0\Pi_2 \ge 0, \quad \Pi_1 + \eta\Pi_2 + \eta^2\Pi_3 \ge 0.$

The main objective of this paper is to design an appropriate sliding surface satisfying some performance requirements. On this basis, a sliding control law is further designed to guarantee that the sliding surface $s(k) = 0$ is reachable in a finite time. Meanwhile, a sufficient condition needs to be found to guarantee the SMDs ISS.

3 Main results

3.1 The stability analysis of the observer error system

First, a Luenberger-like full-order observer is given by:

$$\hat{x}(k+1) = A_\varepsilon\hat{x}(k) + B_{u\varepsilon}\varphi(u) + L(y - C\hat{x}), \tag{5}$$

$$\hat{y}(k) = C\hat{x}(k), \tag{6}$$

where $\hat{x} = (\hat{x}_1^T, \hat{x}_2^T)^T \in \mathbb{R}^n$ is the reconstructed state of vector. $\hat{y}$ is the observer output vector. L is an observer gain matrix, $L = (L_1^T\ L_2^T)^T$.

Remark 1 L is an undetermined unknown matrix with appropriate dimension. It should be noted that the value of L is not unique and depends on the coefficient of the system.

Let $e(k) = x(k) - \hat{x}(k)$, $z(k) = y(k) - \hat{y}(k)$. Then, the observer error system can be described by

$$e(k+1) = (A_\varepsilon - LC + \Delta A)e(k) + \Delta A\hat{x}(k) + D_{w\varepsilon}w(k), \tag{7}$$

$$z(k) = Ce(k). \tag{8}$$

Next, based on LMI and the Lyapunov method, we give a sufficient condition to ensure that the observer error system is ISS with the given $H\infty$ performance index γ.

Theorem 1 *For the observer error system* (7)–(8), *if there exist scalars* $\delta_i > 0$ $(i = 1,2,3,4)$ *and* $\sigma > 0$, *matrices* $P > 0$ *and* Y, *such that the following condition holds*:

$$\phi = \begin{pmatrix} \Lambda_{11} & \Lambda_{12} & P & 0 & 0 & A^TP - C^TY^T \\ * & \Lambda_{13} & 0 & -M^TP\bar{D}_{w2} & 0 & M^TP \\ * & * & -3/2I & 0 & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{D}_{w2}^T PM & \bar{D}_{w2}^T P \\ * & * & * & * & -(\delta_3+\delta_4)I & 0 \\ * & * & * & * & * & -P \end{pmatrix} < 0, \tag{9}$$

where

$$\Lambda_{11} = \bar{A}_1^T(YC) + (YC)^T\bar{A}_1 + (\delta_1+\delta_3)N^TN + \sigma^4/2I + C^TC - A_1^TPA_1 - P_2,$$

$$\Lambda_{12} = P\bar{M}_1 - A_1^TP\bar{M}, \qquad \Lambda_{13} = -(\delta_1+\delta_2)I - M^TPM,$$

$$A_1 = \begin{pmatrix} A_{11} & A_{12} \\ 0 & 0 \end{pmatrix}, \qquad \bar{A}_1 = \begin{pmatrix} A_{11}-I & A_{12} \\ 0 & 0 \end{pmatrix}, \qquad P = \begin{pmatrix} P_{11} & 0 \\ 0 & P_{22} \end{pmatrix},$$

$$P_2 = \begin{pmatrix} 0 & 0 \\ 0 & P_{22} \end{pmatrix}, \qquad \bar{M}_1 = \begin{pmatrix} M_1 \\ 0 \end{pmatrix}, \qquad \bar{D}_{w2} = \begin{pmatrix} 0 \\ D_{w2} \end{pmatrix}, \qquad Y = \begin{pmatrix} Y_1 \\ Y_2 \end{pmatrix}.$$

Then, there exists a scalar $\tilde{\varepsilon} > 0$ such that observer error system is ISS with the given $H\infty$ performance index γ for any $\varepsilon \in (0, \tilde{\varepsilon}]$. In addition, the observer gain matrix is given with the following form:

$$L = P^{-T}Y. \tag{10}$$

Proof Substituting (10) into (9) and then use the Schur's Complement, the following inequality can be derived.

$$\begin{pmatrix} \tilde{\Lambda}_{11} & \tilde{\Lambda}_{12} & P & 0 & 0 \\ * & \tilde{\Lambda}_{22} & 0 & -M^TP\bar{D}_{w2} & 0 \\ * & * & -2/3I & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{D}_{w2}^T PM \\ * & * & * & * & \tilde{\Lambda}_{44} \end{pmatrix} + \begin{pmatrix} (A-LC)^T \\ M^T \\ 0 \\ \bar{D}_{w2}^T \\ 0 \end{pmatrix} P \begin{pmatrix} (A-LC)^T \\ M^T \\ 0 \\ \bar{D}_{w2}^T \\ 0 \end{pmatrix}^T < 0, \tag{11}$$

$$\tilde{\Lambda}_{11} = \bar{A}_1^TP^T(LC) + (LC)^TP\bar{A}_1 + (\delta_1+\delta_3)N^TN + \sigma^4/2I + C^TC - A_1^TPA_1 - P_2,$$

$$\tilde{\Lambda}_{12} = PM_1 - A_1^TPM, \qquad \tilde{\Lambda}_{22} = -(\delta_1+\delta_2)I, \qquad \tilde{\Lambda}_{44} = -(\delta_3+\delta_4)I.$$

Since $P_{11} > 0$ and $P_{22} > 0$, there exists a scalar $\varepsilon_{11} > 0$ satisfying $P_{11} - \varepsilon_{11}^2 P_{21} P_{22}^{-1} P_{21}^T \geq 0$ for all $\varepsilon \in (0, \varepsilon_{11}]$. According to Schur's complement,

$$P_\varepsilon = \begin{pmatrix} P_{11} & \varepsilon P_{21} \\ \varepsilon P_{21}^T & P_{22} \end{pmatrix} > 0, \qquad \varepsilon \in (0, \varepsilon_{11}].$$

Define the Lyapunov function with the following form

$$V_1(e) = e^T P_\varepsilon e.$$

Let $w(k) = 0$, we have

$$\begin{aligned}
\Delta V_1(e) &= e(k+1)^T P_\varepsilon e(k+1) - e(k)^T P_\varepsilon e(k) \\
&= \big((A_\varepsilon - LC + \Delta A)e(k) + \Delta A \hat{x}(k)\big)^T P_\varepsilon \big((A_\varepsilon - LC + \Delta A)e(k) + \Delta A \hat{x}(k)\big) \\
&\quad - e^T P_\varepsilon e \\
&\leq e^T(k)\big\{(A_\varepsilon - LC)^T P_\varepsilon (A_\varepsilon - LC) + 1/\delta_1 (A_\varepsilon - LC)^T P_\varepsilon M M^T P_\varepsilon (A_\varepsilon - LC) \\
&\quad + 1/\delta_2 (A_\varepsilon - LC)^T P_\varepsilon M M^T P_\varepsilon (A_\varepsilon - LC) + \delta_1 N^T N \\
&\quad + 3/2 P_\varepsilon^T P_\varepsilon + \sigma^4/2I - P_\varepsilon\big\} e(k) \\
&\quad + \hat{x}^T(k)\big\{\delta_2 N^T N + \sigma^4 I\big\}\hat{x}(k) \\
&= e^T(k)\big(\bar{\phi}_0 + \bar{\phi}_1 + \varepsilon\bar{\phi}_2 + \varepsilon^2\bar{\phi}_3\big)e(k) + \hat{x}^T(k)\big\{\delta_2 N^T N + \sigma^4 I\big\}\hat{x}(k),
\end{aligned} \tag{12}$$

where

$$\bar{\phi}_0 = \begin{pmatrix} \bar{\Lambda}_{11} & PM_1 - A_1^T PM & P \\ * & -(\delta_1 + \delta_2) - M^T PM & 0 \\ * & * & -2/3I \end{pmatrix},$$

$$\bar{\phi}_1 = \begin{pmatrix} (A - LC)^T \\ M^T \\ 0 \end{pmatrix} P \begin{pmatrix} (A - LC)^T \\ M^T \\ 0 \end{pmatrix}^T,$$

$$\bar{\phi}_2 = \begin{pmatrix} \bar{\Sigma}_{11} & A_1^T PM - (E_0 + A_2 - LC)^T P_3 M & P_3 \\ * & 0 & 0 \\ * & * & 0 \end{pmatrix},$$

$$\bar{\phi}_3 = \begin{pmatrix} (A - \bar{L}_2 C)^T \\ \bar{M}_2^T \\ 0 \end{pmatrix} P_4 \begin{pmatrix} (A - \bar{L}_2 C)^T \\ \bar{M}_2^T \\ 0 \end{pmatrix}^T,$$

$$\bar{\Lambda}_{11} = \bar{A}_1^T P^T (LC) - (LC)^T P \bar{A}_1 + \delta_1 N^T N + \sigma^4/2I - A_1^T P A_1 - P_2,$$

$$\bar{\Sigma}_{11} = (E_0 + A_2 - LC)^T P_1 (A - \bar{L}_2 C) + (A - \bar{L}_2 C)^T P_1^T (E_0 + A_2 - LC) - P_3,$$

$$P_1 = \begin{pmatrix} P_{11} & 0 \\ P_{21}^T & 0 \end{pmatrix}, \qquad P_3 = \begin{pmatrix} 0 & P_{21} \\ P_{21}^T & 0 \end{pmatrix}, \qquad P_4 = \begin{pmatrix} P_{11} & P_{21} \\ P_{21}^T & 0 \end{pmatrix},$$

$$\bar{M}_2 = \begin{pmatrix} 0 \\ M_2 \end{pmatrix}, \qquad A_2 = \begin{pmatrix} 0 & 0 \\ A_{21} & A_{22} \end{pmatrix}, \qquad \bar{L}_2 = \begin{pmatrix} 0 \\ L_2 \end{pmatrix},$$

where $\bar{\phi}_0 < 0$ can be guaranteed by (11). Thus, there exists a scalar $\varepsilon_{12} > 0$ such that $\bar{\phi}_0 + \bar{\phi}_1 + \varepsilon\bar{\phi}_2 + \varepsilon^2\bar{\phi}_3 < 0$ for any given $\varepsilon \in (0, \varepsilon_{12}]$. Note $\varepsilon_1^* = \min\{\varepsilon_{11}, \varepsilon_{12}\}$. Let $a = \lambda_{\min}(-\bar{\phi}_0 - \bar{\phi}_1 - \varepsilon\bar{\phi}_2 - \varepsilon^2\bar{\phi}_3)$, then $a > 0$ for all $\varepsilon \in (0, \varepsilon_1^*]$. Thus,

$$\Delta V_1(x) \le -a\|e(k)\|^2 + \varsigma_1\|\hat{x}(k)\|^2 \le -a(1-\theta)\|e(k)\|^2, \quad \forall \|e(k)\| \ge \frac{\sqrt{4\varsigma_1 a\theta}}{2a\theta}\|\hat{x}(k)\|,$$

where

$$0 < \theta < 1, \qquad \varsigma_1 = \|\delta_2 N^T N + \sigma^4 I\|.$$

The above shows that the observer error system is ISS with respect to estimate state $\hat{x}$ for any $\varepsilon \in (0, \varepsilon_1^*]$.

Next, we consider the $H\infty$ performance of the closed-loop system under the condition $w(k) \in L_2$.

$$\begin{aligned} &V\big(e(k+1)\big) - V\big(e(k)\big) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) \\ &\quad = \begin{pmatrix} e^T(k) & w^T(k) \end{pmatrix} \big(\phi_0 + \phi_1 + \varepsilon\phi_2 + \varepsilon^2\phi_3\big) \begin{pmatrix} e^T(k) & w^T(k) \end{pmatrix}^T \\ &\qquad + \hat{x}^T(k)\big\{(\delta_2 + \delta_4)N^T N + \sigma^4 I\big\}\hat{x}(k), \end{aligned} \tag{13}$$

where

$$\begin{aligned} &e^T(k)(A_\varepsilon - LC)^T P_\varepsilon \Delta A w(k) + e^T(k)\Delta A^T P_\varepsilon (A_\varepsilon - LC)w(k) \\ &\quad \le \delta_3 e^T(k)N^T N e(k) + 1/\delta_3 w^T(k)(A_\varepsilon - LC)^T P_\varepsilon M M^T P_\varepsilon^T (A_\varepsilon - LC)w(k) \\ &\qquad \times \hat{x}^T(k)(A_\varepsilon - LC)^T P_\varepsilon \Delta A w(k) + w^T(k)\Delta A^T P_\varepsilon (A_\varepsilon - LC)\hat{x}(k) \\ &\quad \le \delta_4 \hat{x}^T(k)N^T N\hat{x}(k) + 1/\delta_4 w^T(k)(A_\varepsilon - LC)^T P_\varepsilon M M^T P_\varepsilon^T (A_\varepsilon - LC)w(k), \\ &\phi_0 = \begin{pmatrix} \tilde{\Lambda}_{11} & PM_1 - A_1^T PM & P & 0 & 0 \\ * & -(\delta_1 + \delta_2)I - M^T PM & 0 & -M^T P\bar{D}_{w2} & 0 \\ * & * & -\frac{2}{3}I & 0 & 0 \\ * & * & * & -\gamma^2 I & \bar{D}_{w2}^T PM \\ * & * & * & * & -(\delta_3 + \delta_4)I \end{pmatrix}, \\ &\phi_1 = \begin{pmatrix} A - LC & M & 0 & \bar{D}_{w2} & 0 \end{pmatrix}^T P \begin{pmatrix} A - LC & M & 0 & \bar{D}_{w2} & 0 \end{pmatrix}, \\ &\phi_2 = \begin{pmatrix} \bar{\Sigma}_{11} & A_1^T PM - (E_0 + A_2 - LC)^T P_3 M & P_3 & (E_0 + A_2 - LC)^T P_1^T M & 0 \\ * & 0 & 0 & 0 & 0 \\ * & * & 0 & 0 & 0 \\ * & * & * & 0 & D_w^T P_1 M_1 \\ * & * & * & * & 0 \end{pmatrix}, \\ &\bar{\Sigma}_{11} = (E_0 + A_2 - LC)^T P_1 (A - \bar{L}_2 C) + (A - \bar{L}_2 C)^T P_1^T (E_0 + A_2 - LC) - P_3, \\ &\phi_3 = \begin{pmatrix} A - \bar{L}_2 C & \bar{M}_2 & 0 & D_w & \tilde{M}_1 \end{pmatrix}^T P_4 \begin{pmatrix} A - \bar{L}_2 C & \bar{M}_2 & 0 & D_w & \tilde{M}_1 \end{pmatrix}, \\ &\tilde{M}_1 = \begin{pmatrix} O & M_1^T \end{pmatrix}^T \end{aligned}$$

there exists a scalar $\varepsilon_{13} > 0$, satisfying $\phi_0 + \phi_1 + \varepsilon\phi_2 + \varepsilon^2\phi_3 < 0$ for any given $\varepsilon \in (0, \varepsilon_{13}]$. Note $\tilde{\varepsilon} = \min\{\varepsilon_{11}, \varepsilon_{12}, \varepsilon_{13}\}$. Let $b = \lambda_{\min}(-\phi_0 - \phi_1 - \varepsilon\phi_2 - \varepsilon^2\phi_3)$, then $b > 0$ for all $\varepsilon \in (0, \tilde{\varepsilon}]$. Thus,

$$\Delta V_1\big(x(k)\big) \le -b\big\|e(k)\big\|^2 + \varsigma\big\|\hat{x}(k)\big\|^2 \le -b(1-\theta)\big\|e(k)\big\|^2, \quad \forall\big\|e(k)\big\| \ge \frac{\sqrt{4\varsigma b\theta}}{2b\theta}\big\|\hat{x}(k)\big\|,$$

where

$$0 < \theta < 1, \quad \varsigma = \big\|(\delta_2 + \delta_4)N^T N + \sigma^4 I\}\big\|,$$

which implies $V(e(k+1)) - V(e(k)) + z^T(k)z(k) - \gamma^2 w^T(k)w(k) < 0$. Furthermore, the $H\infty$ performance index under zero initial condition can be given by

$$\begin{aligned} J(k) &= \sum_{k=0}^{\infty} z^T(k)z(k) - \gamma^2 w^T(k)w(k) \\ &\le \sum_{k=0}^{\infty} z^T(k)z(k) - \gamma^2 w^T(k)w(k) + \Delta V_1(k) \\ &= V\big(e(\infty)\big) - V\big(e(0)\big) + \sum_{k=0}^{\infty} z^T(k)z(k) - \sum_{k=0}^{\infty} \gamma^2 w^T(k)w(k) \end{aligned}$$

thus, we have

$$\sum_{k=0}^{\infty} z^T(k)z(k) \le \sum_{k=0}^{\infty} \gamma^2 w^T(k)w(k). \tag{14}$$

It is obvious that the $H\infty$ performance index can be satisfied. □

Remark 2 The estimate of the upper bound $\tilde{\varepsilon}$ for SPSs is an interesting topic. Extensive results for the problem have been proposed [9, 28]. Among them, solving the GEVP is a most common way due to its easier operability and higher accuracy. However, in view of the particularity of inequality (15), the upper bound $\tilde{\varepsilon}$ cannot be obtained directly by solving the following GEVP. Based on Lemma 2.3, we will resolve the problem in a special way. See Theorem 2.

Theorem 2 *After deriving the observer gain matrix L from* (8)–(9), *if there exist matrices* $P_{11} > 0$, $P_{22} > 0$ *and* W_1 *satisfying*:

$$\begin{pmatrix} P_{11} & W_1^T \\ W_1 & P_{22} \end{pmatrix} > 0, \qquad W_1 < \varepsilon P_{21}, \qquad \phi_0 < 0, \qquad \phi_0 + \phi_1 + \varepsilon\phi_2 + \varepsilon^2\phi_3 < 0, \tag{15}$$

where ϕ_0, ϕ_1, ϕ_2, ϕ_3 *are defined in Theorem* 1, *then the observer error system is ISS with respect to estimate-state with disturbance attenuation level* γ *for any* $\varepsilon \in (0, \tilde{\varepsilon}]$.

To obtain the minimum of disturbance attenuation level γ, a possible approach is to transform the minimum problem into an optimization problem, which can be solved with MATLAB.

3.2 Designing scheme of the sliding surface and reachability analysis

In this section, we first construct a sliding surface, under which the sliding mode control law is derived such that the sliding surface is reachable.

First, an observer-based sliding surface function is considered as:

$$s(k) = \sigma(k) + B_{u\varepsilon}^T \hat{x}(k), \tag{16}$$

with

$$\Delta\sigma(k) = B_{u\varepsilon}^T B_{u\varepsilon} K\hat{x}(k) - B_{u\varepsilon}^T A_\varepsilon \hat{x}(k) + B_{u\varepsilon}^T \hat{x}(k),$$

where $B_{u\varepsilon}^T B_{u\varepsilon}$ is non-singular. The gain matrix K is to be obtained later.

$$s(k+1) - s(k) = \left(B_{u\varepsilon}^T B_{u\varepsilon} K\right)\hat{x}(k) + B_{u\varepsilon}^T B_{u\varepsilon}\varphi(u) + B_{u\varepsilon}^T L(y - C\hat{x}).$$

By further derivation, the equivalent control law can be obtained as follows

$$\varphi_{eq} = -K\hat{x}(k) - \left(B_{u\varepsilon}^T B_{u\varepsilon}\right)^{-1} B_{u\varepsilon}^T L\left(y - C\hat{x}(k)\right), \tag{17}$$

let's substitute (17) into observer system (5), the SMDs is given by

$$\hat{x}(k+1) = (A_\varepsilon - B_{u\varepsilon}K)\hat{x}(k) + \tilde{B}L\left(y - C\hat{x}(k)\right), \tag{18}$$

where $\tilde{B} = I - B_{u\varepsilon}(B_{u\varepsilon}^T B_{u\varepsilon})^{-1} B_{u\varepsilon}^T$.

The sliding mode control law is designed as follows:

$$u(k) = -\frac{\vartheta(\hat{x})}{\alpha\|s(k)\|} s(k), \tag{19}$$

with $\vartheta(\hat{x}) = \beta + (\|K\hat{x}(k)\| + \|(B_u^T B_u)^{-1} B_u^T L(y - C\hat{x}(k))\|)$, where $\beta > 0$.

The following theorem guarantees that the sliding surface is reachable.

Theorem 3 *With the Assumption 2.1 and the control law (19), the trajectories of the observer system (5) are driven onto the sliding surface $s(k) = 0$ in a finite time.*

Proof Based on the Assumption 2.1, the matrix $B_{u\varepsilon}^T B_{u\varepsilon}$ is non-singular. To verify the reachability, we define the following Lyapunov function:

$$\begin{aligned}
V_3(k) &= \frac{1}{2} s^T(k)\left(B_u^T B_u\right)^{-1} s(k), \\
\Delta V_3(k) &= \frac{1}{2}\Delta s^T(k)\left(B_{u\varepsilon}^T B_{u\varepsilon}\right)^{-1} s(k+1) + \frac{1}{2} s^T(k)\left(B_{u\varepsilon}^T B_{u\varepsilon}\right)^{-1}\Delta s(k) \\
&\le \frac{1}{2}\varphi^T(u)s(k) + \frac{1}{2}\left\{\left\|\left(K\hat{x}(k)\right)^T\right\| + \left\|\left[\left(B_{u\varepsilon}^T B_{u\varepsilon}\right)^{-1} L\left(y - C\hat{x}(k)\right)\right]^T\right\|\right\}\left\|s(k)\right\| \\
&\quad + \frac{1}{2} s^T(k)\varphi(u) + \frac{1}{2}\left\|s^T(k)\right\|\left\{\left\|K\hat{x}(k)\right\| + \left\|\left(B_{u\varepsilon}^T B_{u\varepsilon}\right)^{-1} L\left(y - C\hat{x}(k)\right)\right\|\right\} \\
&\le -\beta\left\|s(k)\right\|.
\end{aligned} \tag{20}$$

Using (19) and the Assumption 2.3, we have

$$\varphi^T(u)u(k) = -\varphi^T(u)\frac{\vartheta(\hat{x})}{\alpha\|s(k)\|}s(k) \geq \alpha u^T(k)u(k), \tag{21}$$

then

$$\varphi^T(u)s(k) \leq -\vartheta(\hat{x})\|s(k)\|, \tag{22}$$

and

$$s^T(k)\varphi(u) \leq -\vartheta(\hat{x})\|s(k)\|. \tag{23}$$

Substituting (22) and (23) into (20) yields

$$\Delta V_3(k) \leq -\beta\|s(k)\| < 0, \quad \text{for } \|s(k)\| \neq 0. \tag{24}$$

This indicates that the sliding surface is reachable. This completes the proof. □

3.3 Sliding mode dynamics analysis

In this subsection, we will propose a criterion to guarantee that the SMDs is ISS with respect to observer error $e(k)$, from which the controller gain matrix K can be solved.

Theorem 4 *There exist a scalar $\bar{\varepsilon} > 0$ such that the SMDs (18) is ISS with respect to observer error $e(k)$ for any $\varepsilon \in (0, \bar{\varepsilon}]$, if there exist matrices $X_{11} > 0$, $X_{22} > 0$ and G, satisfying*

$$\psi = \begin{pmatrix} X^T\bar{A}^T + \bar{A}X - B_uG - G^TB_u^T & X^T\bar{A}_2^T - G^TB_{u2}^T \\ * & -X_{22} \end{pmatrix} < 0, \tag{25}$$

where

$$X = \begin{pmatrix} X_{11} & 0 \\ X_{21} & X_{22} \end{pmatrix}, \qquad \bar{A} = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} - I \end{pmatrix}, \qquad \bar{A}_2 = \begin{pmatrix} A_{21} & A_{22} - I \end{pmatrix},$$

moreover, the gain matrix is obtained by

$$K = GX^{-1}. \tag{26}$$

Proof For (25), the following inequality is obtained by the Schur's complement

$$\begin{pmatrix} \bar{A}X + X^TA^T - B_uKX - X^TK^TB_u^T & X^T\bar{A}_2 - X^TK^TB_{u2}^T \\ * & -X_{22} \end{pmatrix} < 0. \tag{27}$$

Pre- and post-multiplying inequality (27) by $\operatorname{diag}(X^{-T}, I)$ and $\operatorname{diag}(X^{-1}, I)$, respectively, and let

$$X^{-1} = Q = \begin{pmatrix} Q_{11} & O \\ Q_{21} & Q_{22} \end{pmatrix},$$

the inequality (28) is given by

$$\begin{pmatrix} Q^T(\bar{A}-B_uK)+(\bar{A}-B_uK)^TQ & (\bar{A}_2-B_{u2}K)^T \\ * & -Q_{22}^{-1} \end{pmatrix} < 0, \tag{28}$$

where $Q_2 = \begin{pmatrix} O & O \\ O & Q_{22} \end{pmatrix}$, the inequality (29) is derived from the (28) by using Schur's complement

$$Q^T(\bar{A}-B_uK)+(\bar{A}-B_uK)^TQ+(\bar{A}_2-B_{u2}K)^TQ_{22}(\bar{A}_2-B_{u2}K) < 0, \tag{29}$$

therefore, there exists a scalar $\varepsilon_{21} > 0$ such that $Q_{11} - \varepsilon Q_{12}^T Q_{22}^{-1} Q_{21} > 0$ for all $\varepsilon \in (0, \varepsilon_{21}]$. Based on Schur's complement lemma, it yields

$$Q_\varepsilon = \begin{pmatrix} \varepsilon^{-1}Q_{11} & Q_{21} \\ Q_{21}^T & Q_{22} \end{pmatrix} > 0.$$

Define the following Lyapunov function:

$$\begin{aligned} V_2(k) &= x^T(k)Q_\varepsilon x(k), \qquad (30) \\ \Delta V_2(k) &= \hat{x}^T(k+1)Q_\varepsilon\hat{x}(k+1) - \hat{x}^T(k)Q_\varepsilon\hat{x}(k) \\ &= \big((A_\varepsilon - B_\varepsilon K)\hat{x}(k) + \tilde{B}(LC)e(k)\big)^T Q_\varepsilon \big((A_\varepsilon - B_\varepsilon K)\hat{x}(k) + \tilde{B}(LC)e(k)\big) \\ &\quad - \hat{x}^T(k)Q_\varepsilon\hat{x}(k) \\ &= \hat{x}^T(\psi_0 + \varepsilon\psi_1)\hat{x} + 2\hat{x}^T(k)(A_\varepsilon - B_\varepsilon K)^T Q_\varepsilon \tilde{B}(LC)e(k) \\ &\quad + e^T(k)(\tilde{B}LC)^T Q_\varepsilon(\tilde{B}LC)e(k), \end{aligned}$$

where

$$\begin{aligned} \psi_0 &= Q^T(\bar{A}-B_uK)+(\bar{A}-B_uK)^TQ+(\bar{A}_2-B_{u2}K)^TQ_{22}(\bar{A}_2-B_{u2}K), \\ \psi_1 &= (A-B_uK)^TQ_3(A-B_uK), \qquad Q_3 = \begin{pmatrix} Q_{11} & Q_{21} \\ Q_{21}^T & 0 \end{pmatrix}. \end{aligned}$$

The inequality (29) indicates that there exist a scalar $\varepsilon_{22} > 0$, satisfying $\psi_0 + \varepsilon\psi_1 < 0$ for all $\varepsilon \in (0, \varepsilon_{22}]$. Note $\bar{\varepsilon} = \min\{\varepsilon_{21}, \varepsilon_{22}\}$, let $d = \lambda_{\min}(-\psi_0 - \varepsilon\psi_1)$, then $d > 0$ for all $\varepsilon \in (0, \bar{\varepsilon}]$. Thus,

$$\begin{aligned} \Delta V_2\big(x(k)\big) &\le -d\|\hat{x}(k)\|^2 + \mu\|\hat{x}(k)\|\|e(k)\| + \nu\|e(k)\|^2 \\ &\le -d(1-\theta)\|\hat{x}(k)\|^2, \quad \forall\|\hat{x}(k)\| \ge \frac{\mu + \sqrt{\mu^2 + 4\nu d\theta}}{2d\theta}\|e(k)\|, \end{aligned}$$

where

$$0 < \theta < 1, \quad \mu = 2\sup_{\varepsilon\in(0,\bar{\varepsilon}]}\big\{\big\|(A_\varepsilon - B_\varepsilon K)^T P_\varepsilon \tilde{B}(LC)\big\|\big\}, \qquad \nu = \big\|(\tilde{B}LC)^T Q_\varepsilon(\tilde{B}LC)\big\|.$$

According to Lemma 2.1, the SMDs is ISS with respect to the observer error $e(k)$. This completes the proof. □

Similar to the method used in Theorem 2, the following theorem gives the method for solving the upper bound.

Theorem 5 *After deriving the controller gain matrix K from (26), if there exist matrices $Q_{11} > 0$, $Q_{22} > 0$ and W, a positive scalar θ, satisfying*

$$\begin{pmatrix} W & Q_{21} \\ Q_{21}^T & Q_{22} \end{pmatrix} > 0, \qquad W < \theta Q_{11}, \qquad \psi_0 < 0, \qquad \psi_1 < -\theta\psi_0, \tag{31}$$

then, the SMDs (17) is ISS with respect to observer error $e(k)$ for any $\varepsilon \in (0, \bar{\varepsilon}]$, where $\bar{\varepsilon} = \theta^{-1}$.

4 Numerical examples

In this section, we present two examples to verify the effectiveness of the obtained results.

Example 1 Consider the following uncertain DTSPS with external disturbance.

$$A = \begin{pmatrix} -0.5 & 0.6 \\ -0.6 & -0.5 \end{pmatrix}, \qquad B_u = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad B_w = \begin{pmatrix} 0.2 \\ 0.1 \end{pmatrix},$$
$$M = N = \begin{pmatrix} -0.02 & -0.06 \\ -0.02 & 0.04 \end{pmatrix},$$
$$C = \begin{pmatrix} 0.05 & 0.01 \end{pmatrix}, \qquad E_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \qquad E_\varepsilon = \begin{pmatrix} \varepsilon & 0 \\ 0 & 1 \end{pmatrix}.$$

We might as well make $F = \left(\begin{smallmatrix} 0.5 & 0 \\ 0 & 0.5 \end{smallmatrix}\right)$, the following results can be derived via solving the LMI (9).

$$P = \begin{pmatrix} 0.3413 & 0 \\ 0 & 0.2188 \end{pmatrix}, \qquad Y = \begin{pmatrix} -6.0374 \\ 1.8511 \end{pmatrix}, \qquad \gamma_{\min} = 0.0017.$$

The observed gain matrix $L = P^{-T}Y = (-17.6892\ 8.4593)^T$. According to Theorem 2, the upper bound of perturbed parameter $\bar{\varepsilon} = 1.7758$ can be obtained by solving the LMI (15).

Furthermore, according to Theorem 4, we obtain the following results.

$$X = \begin{pmatrix} 1.7502 & 0 \\ 0.7544 & 1.0996 \end{pmatrix}, \qquad G = \begin{pmatrix} 0.2484 & -1.2169 \end{pmatrix},$$
$$K = GX^{-1} = \begin{pmatrix} 0.6189 & -1.1067 \end{pmatrix}.$$

Furthermore, we get the upper bound $\tilde{\varepsilon} = 4.0535$ by solving GEVP (31). Moreover, when the small parameter ε is assumed to be 0.01, given the initial conditions $x(0) = (0.5\ -15)^T$, $\hat{x}(0) = (1\ -1)^T$, the external disturbance $w(k) = 1/(1 + t^3)$, and the nonlinear disturbance $\varphi(u(k)) = u \cdot \cos u$. Then, the simulation results are presented by the following graphs. Figure 1 shows the responses of the closed-loop system. The responses of the observer

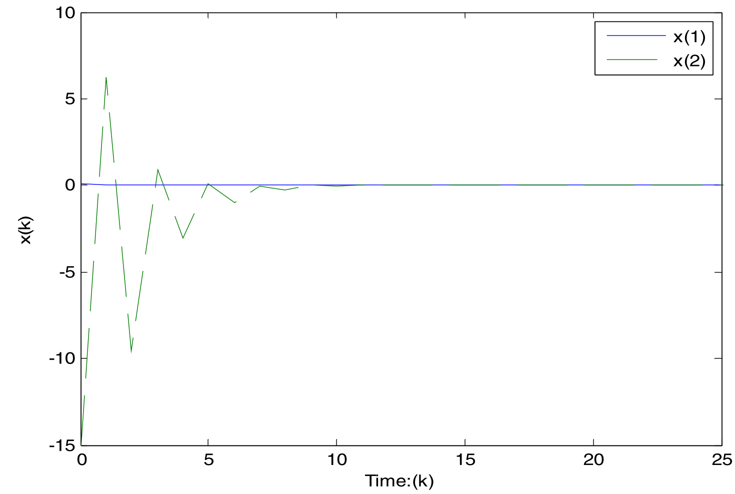

Figure 1 States of closed-loop system

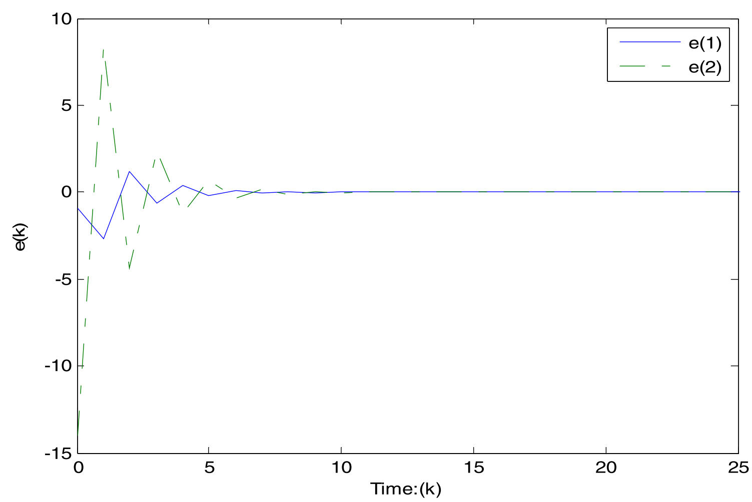

Figure 2 States of observer error system

error system are depicted in Fig. 2. From these simulations, it is clear that the proposed observer-based sliding mode control strategy efficiently reduces the impacts of nonlinearities and ensures the closed-loop system's asymptotic stability. Moreover, the response of the sliding mode variable $s(k)$ is given in Fig. 3, which shows that the sliding motion in the sliding surface $s(\hat{x}(t)) = 0$ can be attained in a finite time.

Example 2 Consider the DTSPS with the following coefficient matrices.

$$A = \begin{pmatrix} -0.2090 & -0.6744 & -0.9126 & 0.1024 \\ 1.4737 & 0.0150 & 0.0203 & 0.0023 \\ -0.0015 & 0 & 0.9886 & 0.0130 \\ 0.0005 & 0 & -0.0625 & 0.9932 \end{pmatrix}, \qquad B = \begin{pmatrix} 0.1432 \\ -16.9432 \\ -0.0022 \\ -0.1139 \end{pmatrix},$$

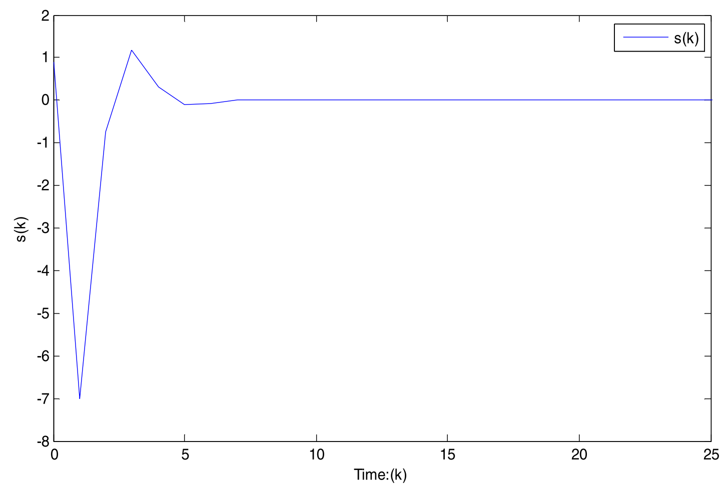

Figure 3 State of the sliding mode variable

$$C = \begin{pmatrix} -0.0500 & 0.0010 & 0.0010 & -0.002 \\ 0.0010 & -0.0010 & 0.0100 & 0.0050 \end{pmatrix}, \qquad D = \begin{pmatrix} 0.1000 & 0 & 0 & 1.0000 \end{pmatrix}^T,$$

$$M = \begin{pmatrix} 0.2 & 0.2 & 0.2 & 0.2 \end{pmatrix}^T, \qquad N = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}.$$

Then, let $F = 0.5$, the following results can be obtained

$$Y = \begin{pmatrix} -1.1789 & -0.1840 \\ 0.0191 & -0.0408 \\ 0.0637 & 0.4720 \\ -0.0228 & 0.2108 \end{pmatrix}, \qquad P = \begin{pmatrix} 0.0603 & -0.0010 & 0 & 0 \\ -0.0010 & 0.0003 & 0 & 0 \\ 0 & 0 & 0.0110 & 0.0000 \\ 0 & 0 & 0.0000 & 0.0026 \end{pmatrix},$$

$$L = P^{-T} Y = \begin{pmatrix} -19.6045 & -5.8739 \\ -2.9772 & -167.4233 \\ 5.8031 & 42.4154 \\ -8.9673 & 81.3245 \end{pmatrix},$$

the minimum of disturbance attenuation level is given by $r_{\min} = 0.0461$. According to Theorem 2, the upper bound of perturbed parameter $\bar{\varepsilon} = 1.4909$ is obtained. Solving LMI (25) in a same manner, the following results are presented

$$G = \begin{pmatrix} 29.8446 \\ -4.1426 \\ 0.2307 \\ 2.4064 \end{pmatrix}, \qquad X = \begin{pmatrix} 59.8528 & 33.4401 & 0 & 0 \\ 33.4401 & 59.6633 & 0 & 0 \\ 46.2038 & 630.2194 & 8.8697 & 2.7035 \\ 630.2194 & 222.9048 & 2.7035 & 51.4828 \end{pmatrix},$$

$$K = GX^{-1} = \begin{pmatrix} 0.3049 & -0.5389 & 0.0120 & 0.0461 \end{pmatrix}.$$

In addition, according to Theorem 5, the upper bound $\tilde{\varepsilon} = 0.0023$ is solved by GEVP.

Next, we choose $\varepsilon = 0.01$, given the initial conditions $x(0) = (-0.05\ 0.75\ -1.5\ 0.12)^T$, $\hat{x}(0) = (-0.3\ -0.3\ -1.9\ 0.5)^T$, the disturbance $w(k) = 1/(1 + k^2)$, and the nonlinear disturbance $\varphi(u(k)) = \cos u^2$. The responses of the closed-loop system and the observer error

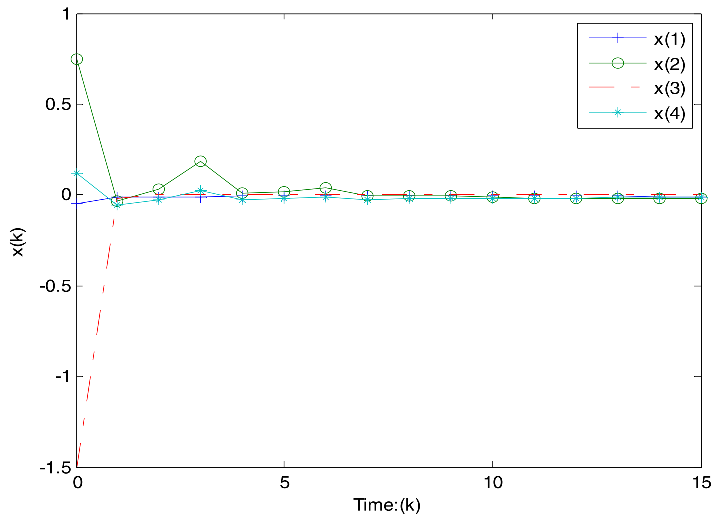

Figure 4 States of closed-loop system

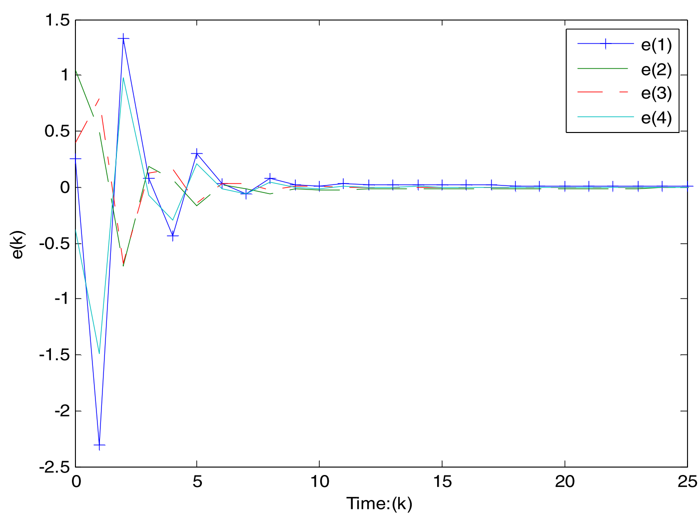

Figure 5 States of observer error system

system are plotted in Figs. 4–5, from which the presented sliding mode control scheme guarantees the asymptotic stability of the closed-loop system. The State of the sliding mode variable $s(k)$ is given in Fig. 6, which demonstrates that the system trajectory is pushed onto the predetermined sliding mode surface in a finite time.

5 Conclusion

The $H\infty$ based on the observer sliding mode control for uncertain DTSPSs has been considered. A suitable SMC law has been designed to make the trajectories of the system tend to the sliding mode surface in a finite time. Based on the Lyapunov method, the observer error system is ensured to be ISS with the given $H\infty$ performance index γ. Meanwhile, the upper bound of the small parameter and the minimum $H\infty$ performance index have been

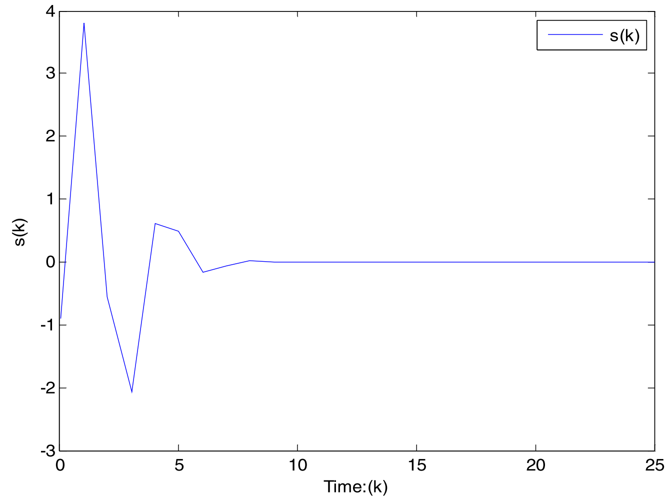

Figure 6 State of the sliding mode variable

obtained in a workable way. Beyond that, the sufficient condition for ISS of the SMDs with regard to the observer error has also been derived in the state estimation space. Finally, two numerical examples have demonstrated the effectively of the provided methods. One of the further research topics is to extend the present results to more general cases, for example, the case that the system is purely nonlinear, the case when the effect of the time delay is considered.

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All data generated or analyzed during this study are included in this published article.

Declarations

Competing interests
The authors declare that they have no competing interests.

Author contributions
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References
1. Wang, Y.Y., Shi, P., Yan, H.C.: Reliable control of fuzzy singularly perturbed systems and its application to electronic circuit. IEEE Trans. Circuits Syst. **65**(10), 1–10 (2018)
2. Martynyuk, A.A., Miladzhanov, V.G.: Stability of singularly perturbed systems with structural perturbations, some applications. Int. Appl. Mech. **39**(8), 875–894 (2003)
3. Siddarth, A., Valasek, J.: Global tracking control structures for nonlinear singularly perturbed aircraft systems. In: Proceedings of the 1st CEAS Specialist Conference on Guidance, Navigation and Control, pp. 235–246 (2017)
4. Zaad, K.H., Khorasani, K.: Control of non-minimum phase singularly perturbed systems with applications to flexible link manipulators. In: Proceedings of Workshop on Advances in Control, pp. 234–256 (1996)
5. Wu, L.G., Zheng, W.X., Gao, H.J.: Dissipativity-based sliding mode control of switched stochastic systems. IEEE Trans. Autom. Control **58**(3), 785–791 (2013)

6. Lacerda, M.J., Seiler, P.: Stability of uncertain systems using Lyapunov functions with non-monotonic terms. Automatica **8**, 183–193 (2017)
7. Wen, C.Y., Zhou, J., Liu, Z.T.: Robust adaptive control of uncertain nonlinear systems in the presence of input saturation and external disturbance. IEEE Trans. Autom. Control **56**(7), 1672–1678 (2011)
8. Asemani, M.H., Majd, V.J.: A robust H∞ non-PDC design scheme for singularly perturbed T-S fuzzy systems with immeasurable state variables. IEEE J. Mag. Trans. Fuzzy Syst. **23**(3), 525–541 (2015)
9. Liu, W., Wang, Y.Y.: Robustness of proper dynamic output feedback for discrete-time singularly perturbed systems. Appl. Math. Comput. **217**(3), 1053–1070 (2017)
10. Yang, C.Y., Zhang, Q.L., Sun, J.: Lur'e Lyapunov function and absolute stability criterion for Lur'e singularly perturbed systems. IEEE Trans. Autom. Control **56**(11), 2666–2671 (2011)
11. Shen, H.H., Hu, X., Wang, J., Cao, J.D., Qian, W.H.: Non-fragile $H\infty$ synchronization for Markov jump singularly perturbed coupled neural networks subject to double-layer switching regulation. IEEE Trans. Neural Netw. Learn. Syst. **34**(5), 2682–2692 (2023). https://doi.org/10.1109/TNNLS.2021.3107607
12. Liu, X.M., Xia, J.W., Wang, J., Shen, H.: Interval type-2 fuzzy passive filtering for nonlinear singularly perturbed PDT-switched systems and its application. J. Syst. Sci. Complex. **34**(6), 2195–2218 (2021)
13. Wu, L.G., Zheng, W.X., Gao, H.J.: Dissipativity-based sliding mode control of switched stochastic systems. IEEE Trans. Autom. Control **58**(3), 785–791 (2013)
14. Wang, J., Yang, C.Y., Xia, J.W., Wu, Z.G., Shen, H.: Observer-based sliding mode control for networked fuzzy singularly perturbed systems under weighted try-once-discard protocol. IEEE Trans. Fuzzy Syst. **30**(6), 1889–1899 (2022)
15. Zhang, H., Liu, X.T., Wang, J.M.: Robust $H\infty$ sliding mode control with pole placement for a fluid power electrohydraulic actuator (EHA) system. Int. J. Adv. Manuf. Technol. **73**(5), 1095–1104 (2014)
16. Wang, Y.Y., Gao, Y.B., Karimi, H.R.: Sliding mode control of fuzzy singularly perturbed systems with application to electric circuit. IEEE Trans. Syst. **48**(10), 1–9 (2017)
17. Lin, K.J.: Adaptive sliding mode control design for a class of uncertain singularly perturbed nonlinear systems. Int. J. Control **87**(2), 432–439 (2014)
18. Liu, W., Wang, Y.Y.: Passivity-based sliding mode control for Lur'e singularly perturbed time-delay systems with input nonlinearity. Circuits Syst. Signal Process. **41**, 6007–6030 (2022)
19. Nagarale, R.M., Patre, B.M.: Composite fuzzy sliding mode control of nonlinear singularly perturbed nonlinear systems. ISA Trans. **53**(3), 679–689 (2014)
20. Yang, C.Y., Che, Z.Y., Fu, J.: Passivity-based integral sliding mode control and ε-bound estimation for uncertain singularly perturbed systems with disturbances. IEEE Trans. Circuits Syst. **14**(8), 1–5 (2018)
21. Ahmed, A.E., Schwartz, H.M., Aitken, V.C.: Sliding mode control for singularly perturbed systems. In: Asian Control Conference, pp. 1946–1950 (2004)
22. Liu, H.W., Zhao, F., Chen, X.Y., Hou, H.Z., Qiu, J.H.: Observer-based finite-time H∞ sliding mode control of stochastic nonlinear singular systems and its applications. Nonlinear Dyn. **108**, 3595–3604 (2022)
23. Wang, G.Z., Zhao, F., Chen, X.Y., Qiu, J.L.: Observer-based finite-time $H\infty$ control of Itô-type stochastic nonlinear systems. Asian J. Control **25**(3), 2378–2387 (2023). https://doi.org/10.1002/asjc.2954
24. Chen, T.T., Hsiao, M.Y., Liu, C.H.: Simplex-type sliding-mode control for a class of linear discrete-time systems. In: Proceedings of SICE Annual Conference, pp. 3062–3065 (2010)
25. Huang, J.J., Li, T.H.: Discrete simplex sliding mode control and its application to singular perturbation systems. IEEE Int. J. Syst. Sci. **48**(4), 818–827 (1996)
26. Zhou, L.N., Che, Z.Y., Yang, C.Y.: Disturbance observer-based integral sliding mode control for singularly perturbed systems with mismatched disturbances. IEEE Access **6**, 9854–9861 (2018)
27. Linares, R.C., Gallegos, J.A., López, V.V.: Sliding mode control and state estimation for a class of nonlinear singularly perturbed systems. Dyn. Control **32**(14), 25–46 (2001)
28. Liu, W., Wang, Y.Y., Wang, Z.M.: $H\infty$ observer-based sliding mode control for singularly perturbed systems with input nonlinearity. Nonlinear Dyn. **85**(1), 573–582 (2017)
29. Liu, Z., Zhu, Q.M., Zhao, L.: A new result on observer-based sliding mode control design for a class of uncertain stochastic delay systems. J. Franklin Inst. **354**(l8), 8200–8216 (2017)
30. Ríos, H., Efimov, D., Perruquetti, W.: An adaptive sliding-mode observer for a class of uncertain nonlinear systems. Int. J. Adapt. Control Signal Process. **32**(3), 511–527 (2018)
31. Chang, J.L.: Discrete-time sliding mode controller design with state estimator and disturbance observer. Electr. Eng. **89**(5), 397–404 (2007)
32. Zhang, J.H., Feng, G., Xia, Y.Q.: Design of estimator-based sliding-mode output-feedback controllers for discrete-time systems. IEEE Trans. Ind. Electron. **61**(5), 2432–2440 (2014)
33. Jiang, Z.P., Wang, Y.: Input-to-state stability for discrete-time nonlinear systems. Automatica **37**, 857–869 (2001)

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