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pth-moment stability of stochastic functional differential equations with Markovian switching and impulsive control

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Abstract

In this paper, we investigate the problem of pth-moment stability of stochastic functional differential equations with Markovian switching and impulsive control via comparison principle. Employing stochastic analysis theory and an impulsive delay differential inequality, we establish a new comparison principle for stochastic functional differential equations with Markovian switching and impulsive control. Using the comparison principle, we derive sufficient conditions for stochastic functional differential equations with Markovian switching and impulsive control by the stability of impulsive delay differential equations. An example is provided to show the effectiveness of the proposed results.

Keywords: pth moment stability; Stochastic functional differential equations; Impulsive; Markovian switching; Comparison principle

1 Introduction

The stochastic functional differential equations with Markovian switching and impulsive control provide very important mathematical modes for many real phenomena and processes in the field of biological and neural networks; see [1–7] and the references therein. In the recent years, pth-moment stability of stochastic functional differential equations with Markovian switching and impulsive control has attracted a considerable attention of researchers in the study of many interesting problems in neural networks, and some criteria of exponential stability for stochastic functional differential equations with Markovian switching and impulsive control have been given [3–7]. For example, Zhu [3], Gao [4], and Kao et al. [5] studied the pth-moment exponential stability of stochastic functional differential equations with Markovian switching and impulsive control by using Lyapunov functionals and Razumikhin technique. Wu et al. [6], discussed the pth-moment exponential stability of stochastic functional differential equations with Markovian switching and impulsive control by using a Razumikhin-type method. Li [7] obtained the pth-moment exponential stability of stochastic functional differential equations with impulsive control.

Motivated by the aforementioned discussions, in the present paper, we further study the comparison principle for stochastic functional differential equations with Markovian switching and impulsive control. It is well known that the comparison principle as an important method has been successfully used in stability analysis for impulsive delay differential equations [8, 9]. In this paper, we first establish a new comparison principle for impulsive stochastic delayed reaction–diffusion equations. Then using this comparison principle, we obtained some stability criteria for stochastic functional differential equations with Markovian switching and impulsive control, such as mean square stability, mean square uniform stability, mean square asymptotical stability, and mean square exponential stability. Therefore the comparison principle proposed in the paper is convenient for the study of stability of stochastic functional differential equations with Markovian switching and impulsive control. We summarize the main contributions of the work as follows:

(1) A novel approach, i.e., a comparison principle is established for stochastic functional differential equations with Markovian switching and impulsive control and impulsive functional differential equation. Meanwhile, bases on the comparison principle, we can easily obtain the pth-moment stability of stochastic functional differential equations with Markovian switching and impulsive control.

(2) Differently from the existing results [3–7], which focus only on the exponential stability for stochastic functional differential equations with Markovian switching and impulsive control, in this work, we obtain some stability criteria for such systems, including the pth-moment stability, pth-moment asymptotic stability, and pth-moment exponential stability.

The rest of the paper is organized as follows. In Sect. 2, we present our mathematical model of stochastic functional differential equations with Markovian switching and impulsive control. Moreover, we give some useful notations and definitions. In Sect. 3, we establish a new comparison principle for stochastic functional differential equations with Markovian switching and impulsive control. In Sect. 4, we give some sufficient conditions for stochastic functional differential equations with Markovian switching and impulsive control by employing the comparison principle. In Sect. 5, we provide an example to illustrate the effectiveness of the obtained results.

2 Model description and preliminaries

Let $\mathbb{R} = (-\infty, +\infty)$ and $Z_+ = \{1, 2, \dots, n\}$, let $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of $n \times m$ real matrices, respectively; $\mathrm{PC}([-\tau, 0]; \mathbb{R}^n)$ stands for the set of piecewise right-continuous functions ψ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the norm $\|\psi\|_\tau = \sup_{-\tau\le\theta\le0} |\psi(\theta)|$. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge t_0}, \mathbb{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\ge t_0}$. By $w(t)$ we denote an m-dimensional Brownian motion on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge t_0}, \mathbb{P})$ and by $\mathrm{PC}^b_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^n)$ the family of $\mathcal{F}_t$-measurable $\mathrm{PC}([-\tau, 0]; \mathbb{R}^n)$-valued stochastic processes $\phi = \{\phi(\theta) : -\tau \le \theta \le 0\}$ such that $\sup_{-\tau\le\theta\le0} \mathbb{E}|\phi(\theta)|^p < \infty$, where $\mathbb{E}$ is the mathematical expectation. Let $r(t)$ be a right-continuous Markov chain defined on $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge t_0}, \mathbb{P})$ and taking values in the finite state space $\mathbb{S} = \{1, 2, \dots, N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$\mathbb{P}\bigl\{\gamma(t+\triangle) = j|\gamma(t) = i\bigr\} = \begin{cases} \gamma_{ij}\triangle + o(\triangle) & \text{if } i \neq j, \\ 1 + \gamma_{ii}\triangle + o(\triangle) & \text{if } i = j, \end{cases} \tag{2.1}$$

where $\triangle > 0$, and $\lim_{\triangle\to0} \frac{o(\triangle)}{\triangle} = 0$. If $j \neq i$, $\gamma_{ij} \ge 0$ is the transition rate from i to j, and $\gamma_{ii} = -\Sigma_{j\neq i}\gamma_{ij}$.

Consider the following stochastic functional differential equations with Markovian switching and impulsive control:

$$\begin{cases} dx(t) = f(t,x(t),x_t,r(t))\,dt + g(t,x(t),x_t,r(t))\,dw(t), & t \neq t_k, \\ x(t_k) = I_k(t_k,x(t_k^-)), & k \in \mathbb{Z}_+, \\ x_{t_0}(\theta) = \phi(\theta) \in \mathrm{PC}^b_{\mathcal{F}_{t_0}}([-\tau,0];\mathbb{R}^n), \end{cases} \tag{2.2}$$

where $x(t) = (x_1(t),x_2(t),\dots,x_n(t))^T$, $x_t = \{x(t+\theta),-\tau \le \theta \le 0,\tau > 0\}$, $f : \mathbb{R}^+ \times \mathbb{R}^n \times \mathrm{PC}([-\tau,0];\mathbb{R}^n) \times \mathbb{S} \to \mathbb{R}^n$ and $g : \mathbb{R}^+ \times \mathbb{R}^n \times \mathrm{PC}([-\tau,0];\mathbb{R}^n) \times \mathbb{S} \to \mathbb{R}^{n\times m}$ are nonlinear functions, and $I_k(t_k,x(t_k^-)) : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}^n$ stands for impulsive perturbations of x at the time t_k. The times t_k represent the impulsive moments satisfying $0 \le t_0 < t_1 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = +\infty$. Moreover, we assume that $f(t,0,0,i) \equiv 0$ and $g(t,0,0,i) \equiv 0$. Then system (2.1) has a trivial solution $x \equiv 0$.

Definition 2.1 A function $V : [t_0 - \tau,+\infty) \times \mathbb{R}^n \times \mathbb{S} \to \mathbb{R}^+$ belongs to the class $\mathcal{V}_0$ if

(i) the function V is continuously differentiable in t and twice differentiable in x in each of the sets $[t_{k-1},t_k) \times \mathbb{R}^n \times \mathbb{S}$, $k \in \mathbb{Z}_+$. In addition, $V(t,0,i) = 0$ for $t \ge t_0$.

(ii) $V(t,x,i)$ is locally Lipschitzian in $x \in \mathbb{R}^n$;

(iii) $\lim_{(t,z,i)\to(t_k^-,x,i)} V(t,z,i) = V(t_k^-,x,i)$, and $\lim_{(t,z,i)\to(t_k^+,x,i)} V(t,z,i) = V(t_k^+,x,i)$.

For any $V \in \mathcal{V}_0$ and $(t,x_t,i) \in [t_k,t_{k+1}) \times \mathrm{PC}^b_{\mathcal{F}_t}([-\tau,0];\mathbb{R}^n) \times \mathbb{S}$, we define the operator $\mathcal{L}V$ as follows:

$$\begin{aligned} \mathcal{L}V(t,x_t,i) = {} & V\bigl(t,x(t),i\bigr) + V_x\bigl(t,x(t),i\bigr)f\bigl(t,x(t),x_t,i\bigr) \\ & + \frac{1}{2}\mathrm{trace}\bigl[g^T\bigl(t,x(t),\phi,i\bigr)V_{xx}g\bigl(t,x(t),x_t,i\bigr)\bigr]. \end{aligned} \tag{2.3}$$

Definition 2.2 The trivial solution $x(t) \equiv 0$ of system (2.2) is said to be

(i) pth-moment stable if for all $\varepsilon > 0$ and $t_0 \in \mathbb{R}_+$, there exists a constant $\delta = \delta(t_0,\varepsilon) > 0$ such that $\mathbb{E}|x(t,\phi)|^p < \varepsilon$ for $t \ge t_0$ whenever $\sup_{-\tau\le\theta\le0} \mathbb{E}|\phi(\theta)|^p \le \delta$.

(ii) pth-moment asymptotically stable if it is stable in mean square and for all $\varepsilon > 0$ and $t_0 \in \mathbb{R}_+$, there exist $\delta > 0$ and $T = T(t_0,\varepsilon) > 0$ such that $\mathbb{E}|x(t,\phi)|^p < \varepsilon$ for $t \ge T + t_0$ whenever $\sup_{-\tau\le\theta\le0} \mathbb{E}|\phi(\theta)|^p \le \delta$.

(iii) pth-moment exponentially stable if there exist two positive constants λ and K such that

$$\mathbb{E}\bigl\|x(t,\phi)\bigr\|^p \le K \sup_{-\tau\le\theta\le0} \mathbb{E}\bigl|\phi(\theta)\bigr|^p e^{-\lambda(t-t_0)}, \quad t \ge t_0.$$

Definition 2.3 A function $b(r)$ is said to belong to the class $\mathcal{K}$ if $b \in C(\mathbb{R}_+,\mathbb{R}_+)$, $b(0) = 0$, and $b(r)$ is strictly increasing in r. A function $b(r)$ is said to belong to the class $\mathcal{VK}$ if it is a $\mathcal{K}$-class convex function. A function $a(t,r)$ is said to belong to the class $\mathcal{CK}$ if $a \in C(\mathbb{R}_+ \times \mathbb{R}_+,\mathbb{R}_+)$, $a(t,0) \equiv 0$, and $a(t,r)$ is concave and strictly increasing in r for each $t \ge t_0 \in \mathbb{R}_+$.

3 Comparison principle

Consider the following impulsive functional differential equation:

$$\begin{cases} y'(t) = h(t, y(t), y_t), & t \neq t_k, t \geq t_0, \\ y(t_k) = H_k(y(t_k^-)), & k \in \mathbb{Z}^+, \\ y_{t_0}(\theta) = \xi(\theta), & \theta \in [-\tau, 0], \end{cases} \tag{3.1}$$

where $y_t(\theta) = y(t+\theta), \theta \in [-\tau, 0]$, the initial value $\xi(\theta)$ is a bounded $\mathrm{PC}([-\tau, 0], \mathbb{R}_+)$-valued function, $H_k : \mathbb{R}_+ \to \mathbb{R}_+$ is nondecreasing, and $H_k(t) \leq t$ for all $k \in \mathbb{Z}_+$. Let $H_k(0) \equiv 0$. Therefore system (3.1) admits a trivial solution $y(t) \equiv 0$. We assume that system (3.1) has a solution for any initial value function ξ.

Theorem 3.1 (Comparison principle) *Assume that there exists a function $V \in \mathcal{V}_0$ satisfying*

(i) $\mathbb{E}\mathcal{L}V(t, x_t, i) \leq h(t, \mathbb{E}V(t, x(t), i), \mathbb{E}V_t)$, $t \geq t_0$, $t \neq t_k$, *where* $V_t = V(t+\theta, x(t+\theta), i)$, $\theta \in [-\tau, 0]$;

(ii) $\mathbb{E}V(t_k, I_k(t_k, x(t_k^-))) \leq H_k(\mathbb{E}V(t_k, x(t_k^-), i))$, $k \in \mathbb{Z}_+$.

Then

$$\mathbb{E}V\big(t, x(t), i\big) \leq r\big(t; t_0, \xi(\theta)\big), \quad t \in [t_0, \infty),$$

provided that $\mathbb{E}V(t_0 + s, x(t_0 + \theta), i) \leq \xi(\theta)$ *for* $\theta \in [-\tau, 0]$, *where* $r(t; t_0, \xi(\theta))$ *is the maximal solution of* (3.1).

Proof For any $t \in [t_{k-1}, t_k)$ and $\Delta t > 0$ sufficiently small satisfying $t + \Delta t < t_k$, from Itô's formula and condition (i) we get

$$\begin{aligned} \mathbb{E}V\big(t + \Delta t, x(t + \Delta t), i\big) - \mathbb{E}V\big(t, x(t), i\big) &\leq \int_t^{t+\Delta t} \mathbb{E}\mathcal{L}V(s, x_t, i)\, ds \\ &\leq \int_t^{t+\Delta t} h\big(s, \mathbb{E}V\big(s, x(s), i\big), \mathbb{E}V_s\big)\, ds. \end{aligned} \tag{3.2}$$

Dividing both sides of Eq. (3.2) by Δt and taking the limit superior as $\Delta t \to 0^+$, we get

$$D^+ m(t) \leq h\big(t, m(t), m_t(\theta)\big), \quad t \in [t_{k-1}, t_k), \tag{3.3}$$

where $m(t) = \mathbb{E}V(t, x(t), i)$, $m_t(\theta) = \mathbb{E}V(t+\theta, x(t+\theta), i)$, and $\theta \in [-\tau, 0]$.

On the one hand, from Theorem 8.1.4 in [10] we can obtain

$$\mathbb{E}V\big(t, x(t), i\big) \leq r\big(t; t_0, \xi(\theta)\big), \quad t \in [t_{k-1}, t_k), \tag{3.4}$$

when $\mathbb{E}V(t_0 + \theta, \phi, i) = \mathbb{E}V(t_0 + \theta, x(t_0 + \theta), i) \leq \xi(\theta)$.

On the other hand, at the impulsive moments t_k, from condition (ii) we get

$$\begin{aligned} \mathbb{E}V\big(t_k, x(t_k), i\big) &= \mathbb{E}V\big(t_k, I_k\big(t_k, x\big(t_k^-\big)\big), i\big) \\ &\le H_k\big(\mathbb{E}V\big(t_k, x\big(t_k^-\big), i\big)\big) \\ &\le H_k\big(r\big(t; t_0, \xi(\theta)\big)\big) \\ &\le r\big(t; t_0, \xi(\theta)\big). \end{aligned} \tag{3.5}$$

Therefore from the above proof we get

$$\begin{cases} D^+ m(t) \le h(t, m(t), m_t), & t \ne t_k, t \in [t_0, \infty), \\ m(t_k) \le H_k(m(t_k^-)), & k \in \mathbb{Z}_+, \\ m_{t_0}(\theta) = \mathbb{E}V(t_0 + \theta, \phi(\theta), i), & \theta \in [-\tau, 0]. \end{cases} \tag{3.6}$$

Obviously, from Theorem 3.1 in [8] we immediately get

$$\mathbb{E}V\big(t, x(t), i\big) \le r\big(t; t_0, \xi(\theta)\big), \quad t \in [t_0, +\infty), \tag{3.7}$$

when $\mathbb{E}V(t_0 + s, x(t_0 + \theta), i) \le \xi(\theta)$. □

4 Stability analysis of stochastic functional differential equations with Markovian switching and impulsive control

Theorem 4.1 *Let conditions* (i) *and* (ii) *of Theorem* 3.1 *hold, and assume that there exist* $V(t, x(t), i) \in \mathcal{V}_0$, $b \in \mathcal{VK}$, *and* $a \in \mathcal{CK}$ *satisfying*

(iii)

$$b\big(\big|x(t)\big|^p\big) \le V\big(t, x(t), i\big) \le a\big(t, \big|x(t)\big|^p\big). \tag{4.1}$$

Then the trivial solution of system (2.2) *is pth-moment stable if the trivial solution of* (3.1) *is stable. The trivial solution of system* (2.2) *is pth-moment asymptotically stable if the trivial solution of* (3.1) *is asymptotically stable.*

Proof Since the trivial solution of (3.1) is stable, for any given $\varepsilon > 0$, there exists $\delta = \delta(t_0, \varepsilon) > 0$ with $\delta < b(\varepsilon)$ such that when $\|\xi\|_\tau < \delta$,

$$\big|y\big(t; t_0, \xi(\theta)\big)\big| < b(\varepsilon) \quad \forall t \ge t_0. \tag{4.2}$$

Since conditions (i) and (ii) of Theorem 3.1 are satisfied, by this theorem we get

$$\mathbb{E}V\big(t, x(t), i\big) \le r\big(t; t_0, \xi(\theta)\big) < b(\varepsilon) \quad \forall t \ge t_0. \tag{4.3}$$

Using condition (iii), we get

$$0 \le b\big(\mathbb{E}\big|x(t)\big|^p\big) \le \mathbb{E}b\big(\big|x(t)\big|^p\big) \le \mathbb{E}V\big(t, x(t), i\big), \quad t \ge t_0. \tag{4.4}$$

Combining (4.3) and (4.4), we get

$$b\bigl(\mathbb{E}\bigl|x(t)\bigr|^p\bigr) \le b(\varepsilon), \quad t \ge t_0. \tag{4.5}$$

Because the function b belongs to $\mathcal{VK}$, we get

$$\mathbb{E}\bigl|x(t)\bigr|^p \le \varepsilon, \quad t \ge t_0. \tag{4.6}$$

Hence we the trivial solution of system (2.2) is pth-moment stable if the trivial solution of (3.1) is stable.

Next, we prove that the trivial solution of system (2.2) is attractive in the pth moment. Since the trivial solution $y(t;t_0,\xi(s))$ of (3.1) is attractive, for any given $\varepsilon > 0$, there exist $\delta_2 = \delta_2(t_0,\varepsilon)$ and $T = T(t_0,\delta_2)$ such that when $\|\xi\|_\tau < \delta_2$,

$$\bigl|y\bigl(t;t_0,\xi(\theta)\bigr)\bigr| < b(\varepsilon), \quad \text{for } t \ge T + t_0. \tag{4.7}$$

Since the function a belongs to $\mathcal{CK}$, we obtain

$$\begin{aligned}\mathbb{E}V\bigl(t_0+\theta, x(t_0+\theta), i\bigr) &\le \mathbb{E}a\bigl(t_0+\theta, \bigl|x(t_0+\theta)\bigr|^p\bigr)\\ &\le a\bigl(t_0+\theta, \mathbb{E}\bigl|x(t_0+\theta)\bigr|^p\bigr) \quad \forall\theta \in [-\tau,0].\end{aligned} \tag{4.8}$$

For any $\theta \in [-\tau,0]$, we set $\xi(\theta) = a(t_0+\theta, \mathbb{E}|x(t_0+\theta)|^p)$. From (4.8) it follows that $\mathbb{E}V(t_0+\theta, x(t_0+\theta), i) \le \xi(\theta)$. Obviously, Theorem 3.1 holds. Consequently, we can find $\delta_3 > 0$ such that, simultaneously,

$$\sup_{-\tau\le\theta\le 0} \mathbb{E}\bigl|x(t_0+\theta)\bigr|^p < \delta_3 \quad \text{and} \quad \|\xi\|_\tau < \delta_2. \tag{4.9}$$

From Eq. (4.7), Eq. (4.8), and Theorem 3.1 we derive that

$$\begin{aligned}b\bigl(\mathbb{E}\bigl|x(t)\bigr|^p\bigr) &\le \mathbb{E}b\bigl(\bigl|x(t)\bigr|^p\bigr) \le \mathbb{E}V\bigl(t,x(t),i\bigr)\\ &\le r\bigl(t;t_0,\xi(s)\bigr) < b(\varepsilon) \quad \forall t \ge T + t_0.\end{aligned}$$

Because the function b belongs to $\mathcal{VK}$, we get

$$\mathbb{E}\bigl|x(t)\bigr|^p < \varepsilon \quad \text{for } t \ge T + t_0. \tag{4.10}$$

Thus the trivial solution of system (2.2) is attractive in the pth moment. By the above discussion the trivial solution of system (2.2) is asymptotically stable in the pth moment if the trivial of solution of (3.1) is asymptotically stable. □

Theorem 4.2 *Let conditions* (i) *and* (ii) *of Theorem* 3.1 *hold, and assume that there exists* $V(t,x(t),i) \in \mathcal{V}_0$ *satisfying*

(iii)′

$$\lambda_1\bigl|x(t)\bigr|^p \le V\bigl(t,x(t),i\bigr) \le \lambda_2\bigl|x(t)\bigr|^p, \tag{4.11}$$

where $\lambda_1 > 0$ *and* $\lambda_2 > 0$.

Then the trivial solution of system (2.2) *is exponentially stable in the pth moment if the trivial solution of* (3.1) *is exponentially stable.*

Proof Since the trivial solution of (3.1) is exponentially stable, there exist two positive constants λ and k such that $|y(t;t_0,\xi(s))| \leq k\|\xi\|_\tau e^{-\lambda t}$ for $t \geq t_0$, which implies that

$$r\big(t;t_0,\xi(s)\big) \leq k\|\xi\|_\tau e^{-\lambda t} \quad \forall t \geq t_0. \tag{4.12}$$

From (4.11) we obtain that

$$\mathbb{E}V\big(t_0+\theta,x(t_0+\theta),i\big) \leq \lambda_2\mathbb{E}\big|x(t_0+\theta)\big|^p \quad \forall \theta \in [-\tau,0]. \tag{4.13}$$

For any $\theta \in [-\tau,0]$, we choose $\xi(\theta) = \lambda_2\mathbb{E}|x(t_0+\theta)|^p = \lambda_2\mathbb{E}|\phi(\theta)|^p$. From (4.13) we obtain $\mathbb{E}V(t_0+\theta,x(t_0+\theta),i) \leq \xi(\theta)$. Obviously, Theorem 3.1 holds.

From Eq. (4.12), Eq. (4.13), and Theorem 3.1 we get

$$\lambda_1\mathbb{E}\big|x(t)\big|^p \leq \mathbb{E}V\big(t,x(t),i\big) \leq r\big(t;t_0,\xi(s)\big) \leq k\lambda_2 e^{-\lambda t}\sup_{-\tau\leq\theta\leq 0}\big(\mathbb{E}\big|\phi(\theta)\big|^p\big), \tag{4.14}$$

from which we get that

$$\mathbb{E}\big|x(t)\big|^p \leq Ke^{-\lambda t}\sup_{-\tau\leq\theta\leq 0}\big(\mathbb{E}\big|\phi(\theta)\big|^p\big), \tag{4.15}$$

where $K = \frac{k\lambda_2}{\lambda_1}$.

Hence from (4.15) we derive that the trivial solution of system (2.2) is exponentially stable in the pth moment. □

5 Example

Example 5.1 Consider the following model:

$$\begin{cases} dx(t) = [A(r(t))x(t) + B(r(t))x(t-\tau)]\,dt + [C(r(t))x(t) + D(r(t))x(t-\tau)]\,dw(t), \\ \quad t \neq t_k, \\ x(t_k) = I_k(x(t_k^-)), \quad t_k \in \mathbb{Z}^+, \\ x(\theta) = \phi(\theta), \quad -\tau \leq \theta \leq 0, \end{cases} \tag{5.1}$$

where $x(t) = (x_1,x_2)^T$, $w(t)$ is a two-dimensional normalized Brownian motion defined on a complete probability space, $\{r(t), t \geq 0\}$ is the Markov process taking values in $\mathbb{S} = \{1,2\}$ with generator $\Gamma = (q_{ij})_{2\times 2}$. $|I_k(x(t_k^-))| \leq \mu_k|x(t_k^-)|$, $\mu_k > 0$.

Choose $V(t,x(t),i) = p_i|x(t)|^2$, where p_i $(i = 1,2)$ are positive constants. Then we have

$$\min\{p_1,p_2\}\big|x(t)\big|^2 \leq V\big(t,x(t),i\big) \leq \max\{p_1,p_2\}\big|x(t)\big|^2, \tag{5.2}$$

$$\begin{aligned}\mathbb{E}\mathcal{L}V\big(t,x(t),i\big) &= 2x^T(x)p_i\big[A(i)x(t)+B(i)x(t-\tau)\big]+\sum_{j=1}^{2}\gamma_{ij}x^Tp_jx(t)\\ &\quad+p_i\big[x^TC^T(i)C(i)x(t)+2x^TD^T(i)D(i)x(t-\tau)\\ &\quad+x^T\big(t-\delta(t)\big)D^T(i)D(i)x(t-\tau)\big]\\ &\le x^Tp_i\left[A^T(i)+A(i)+\left(1+\frac{1}{p_i}\sum_{j=1}^{2}q_{ij}p_j\right)E+2C^T(i)C(i)\right]x(t)\\ &\quad+x^T(t-\tau)p_i\big[B^T(i)B(i)+2D^T(i)D(i)\big]x(t-\tau).\end{aligned} \tag{5.3}$$

Namely,

$$\begin{aligned}&\max_{1\le i\le 2}\mathbb{E}\mathcal{L}V\big(t,x(t),i\big)\\ &\quad\le\frac{\max\{\lambda_{\max}(H(1)),\lambda_{\max}(H(2))\}}{\min\{p_1,p_2\}}\min_{1\le i\le 2}\mathbb{E}V\big(t,x(t),i\big)\\ &\qquad+\frac{\max\{\lambda_{\max}(G(1)),\lambda_{\max}(G(2))\}}{\min\{p_1,p_2\}}\min_{1\le i\le 2}\mathbb{E}V\big(t-\tau,x(t-\tau),i\big),\end{aligned} \tag{5.4}$$

where $H(i)=p_i[A^T(i)+A(i)+(1+\frac{1}{p_i}\sum_{j=1}^{2}q_{ij}p_j)E+2C^T(i)C(i)]$, and $G(i)=p_i[B^T(i)B(i)+2D^T(i)D(i)]$ $(i=1,2)$.

On the other hand,

$$\begin{aligned}\mathbb{E}V\big(t,x(t),i\big)&\le\max\{p_1,p_2\}\big|x(t_k)\big|^2=\max\{p_1,p_2\}\big|I_k\big(x\big(t_k^-\big)\big)\big|^2\\ &\le\max\{p_1,p_2\}\mu_k^2\big|x\big(t_k^-\big)\big|^2\le\frac{\max\{p_1,p_2\}\mu_k^2}{\min\{p_1,p_2\}}\mathbb{E}V\big(t_k^-,x\big(t_k^-\big),i\big).\end{aligned} \tag{5.5}$$

Let $a=-\frac{\max\{\lambda_{\max}(H(1)),\lambda_{\max}(H(2))\}}{\min\{p_1,p_2\}}$, $b=\frac{\max\{\lambda_{\max}(G(1)),\lambda_{\max}(G(2))\}}{\min\{p_1,p_2\}}$, and $c=\frac{\max\{p_1,p_2\}\mu_k^2}{\min\{p_1,p_2\}}$. Thus the comparison system of system (5.1) is

$$\begin{cases}y'(t)=-ay(t)+by(t-\tau), & t\ge t_0, t\ne t_k,\\ y(t_k)=cy(t_k^-), & k\in\mathbb{Z}_+,\\ y_{t_0}=\xi(\theta), & \theta\in[-\tau,0].\end{cases} \tag{5.6}$$

By Corollary 3.2 in [11] we obtain that system (5.6) is exponentially stable if the following conditions are satisfied:

$$t_k-t_{k-1}\ge\mu,\quad |b|<a,\quad \max\big\{1,c^2\big\}<e^{\lambda\mu}, \tag{5.7}$$

where λ satisfies $\lambda-2c+|b|+|b|e^{\lambda\tau}$.

By Theorem 4.2 we conclude that the trivial solution of system (5.1) is exponentially stable in mean square if (5.7) holds.

6 Conclusion

In the paper, we have established a comparison principle to study the pth-moment stability of stochastic functional differential equations with Markovian switching and impulsive control. The established comparison principle can be applied to investigate stability of stochastic functional differential equations with Markovian switching and impulsive control. We can see from our theorems and an example that sufficient conditions are easily obtained by employing the comparison principle together with the stability criteria for impulsive functional differential equations. We believe that the comparison principle is quite general and can be used to analyze other important problems.

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Declarations

Competing interests
The author declares no competing interests.

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